Assignment 1

Grammars & Truth Tables

1. The grammar:

N F | F [e] S I

F I [.] I

I D | D I

D [0] | [1] | [2] | ... | [9]

S [+] | [-]

1. 6.02e+23

N

F [e] S I

I [.] I [e] S I

D [.] I [e] S I

D [.] D I [e] S D I

D [.] D D [e] S D I

D [.] D D [e] S D D

D [.] D D [e] [+] D D

6 [.] D D [e] [+] D D

6 . D D [e] [+] D D

6 . 0 D [e] [+] D D

6 . 0 2 [e] [+] D D

6 . 0 2 e [+] D D

6 . 0 2 e + D D

6 . 0 2 e + 2 D

6 . 0 2 e + 2 3

1. 1e-6

It is impossible to otainn this number with the given grammar. In order to be able to derive this number, the grammar must be modified as follows:

N F | F [e] S I

F I | I [.] I

I D | D I

D [0] | [1] | [2] | ... | [9]

S [+] | [-]

The rule F I has been added so as to represent values that do not contain the „.” character. Using this grammar, the value 1e-6 can be represented as such:

N

F [e] S I

I [e] S I

D [e] S I

D [e] S D

D [e] [-] D

1 [e] [-] D

1 e [-] D

1 e - D

1 e - 6

1. Grammar for formulae in propositional logic:

F Ap | F | F F | F F | F F

Ap [P] | [Q] | [R]

In order to represent arguments in propositional logic, expressed using sequent notation, this grammar needs to be extended as follows:

Starting symbol: S

S F O F | F

F Ap | F | F F | F F | F F | F [‚] F

Ap [P] | [Q] | [R]

O [:] | [**˫**]

* Added the new starting symbol S with the rule S F O F | F such that we are able to represent coherent, complete arguments.
* Added the rule F F [‚] F such that we can enumerate multiple premises if needed.
* Added the symbol O with the rule O [:] | [**˫**] to allow expressing wheter anargument is knowingly valid.

1. If Alice studies logic, then Bob studies it too. A B

If either Alice or Bob studies logic, then Alice definitely does. (A B) A

Therefore, both Alice and Bob study logic. A B

Atomic propositions:

* Alice studies logic. A
* Bob studies logic. B

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | A B | A B | (A B) A | A B |
| T | T | T | T | T | T |
| T | F | T | F | F | F |
| F | T | T | T | T | F |
| F | F | F | T | T | F |

The argument is invalid. From the truth table, we can observe that there are cases when both premises are true but the conclusion is false.

1. P Q, Q R : (P Q) R

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Q | R | P Q | (P Q) | P Q | Q R | (P Q) R |
| T | T | T | T | F | T | T | T |
| T | T | F | T | F | T | T | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | F |
| F | T | T | F | T | T | T | T |
| F | T | F | F | T | T | T | F |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | F | F |

From the truth table we learn that the argument is invalid as it there are cases when both presmises are true but the conclusion is false.

A propositional logic formula that includes atomic propositions P and Q (but not R)

and which, when added as a premise to this argument, makes it valid is Q P.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P | Q | R | P Q | (P Q) | Q P | P Q | Q R | (P Q) R |
| T | T | T | T | F | T | T | T | T |
| T | T | F | T | F | T | T | T | T |
| T | F | T | F | T | T | F | T | T |
| T | F | F | F | T | T | F | F | F |
| F | T | T | F | T | F | T | T | T |
| F | T | F | F | T | F | T | T | F |
| F | F | T | F | T | T | T | T | T |
| F | F | F | F | T | T | T | F | F |

This way, everytime the presmises are true, the conclusion is true as well, having added a premise that is false in the case where all the other premises were true and the conclusion was false.